Modal PML

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***Abstract*—Hybrid numerical techniques in time domain offer computationally efficient means of analysis of certain classes of microwave structures. One of the recently proposed techniques combines the finite-difference time-domain (FDTD) method with the eigenfunction expansion. This method has proven to be very efficient in the analysis of properties of complex planar transmission lines and waveguide discontinuity problems. To achieve full functionality, in particular in the context of the discontinuity analysis, this method has to be complemented by a suitable high-performance absorbing boundary condition. In this letter, we examine a modified Berenger's perfectly matched layer (PML). Tests in a waveguide indicate that low reflections can be obtained in a wide frequency range with few layers of the absorbing medium with a properly selected conductivity profile.**

I. Introduction

A HYBRID finite-difference time-domain (FDTD) eigenfunction expansion technique utilizes a standard FDTD mesh in the areas of structure inhomogeneouity and expansion of fields into a known set of modes in transversely homogeneous regions of the structure [1]–[5]. For example, in the case of a discontinuity in a rectangular waveguide, the discontinuity area is analyzed using the FDTD, and in the remaining areas the fields are represented by only a few orthogonal modes. This provides substantial savings, both in the required computer storage and in the CPU time, at the expense of narrowing the class of problems that can be successfully simulated. The acceleration factor in the computations of transmission line properties can be as high as 40 [1]–[3], as the discretization is performed in one rather than two dimensions. Similarly, in the computations of waveguide discontinuity problems, the discretization is reduced from two dimensions (2-D) to one-dimensional (1-D) or from three-dimensional (3-D) to 1-D, depending on the nature of the discontinuity and the set of modes that can be excited, as the structure is divided into slices with modes representing the unknown fields in each slice. If one could absorb higher-order modes close to discontinuity, the speed of computation would further increase. Also, in many cases, at least one boundary of the structure where the modal expansion is used have to be terminated with a reflectionless load. One efficient and fast solution to this problem, particularly if the evanescent modes has to be absorbed, is to use digital filters based on the Laguerre [6] or even better Kautz [7] polynomials. Another option, useful for guided waves, is the recently introduced Berenger's perfectly matched layer (PML) [8], which offers excellent absorbing performance for the classical FDTD and FEM methods. The formulation and performance of the PML modified for modal expansion are explored in this letter.

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II. Formulation

To terminate the open spaces in the hybrid scheme, suitable boundary conditions that are formulated for modes rather than fields as in FDTD are required. Berenger's PML medium seems particularly well suited, as it provides excellent absorption for guided waves. Since the field dependence in the transverse direction (in each slice) is known (modal solution), Maxwell's equations governing the field behavior become substantially simplified. Accordingly, Berenger's split field formulas reduce to (only four representative equations are given; the remaining having analogical form)

$$\mu_0 \frac{d}{dt} H_{zx} + \sigma_x^* H_{zx} = -K_x^m E_y \tag{1}$$

$$\mu_0 \frac{d}{dt} H_{xz} + \sigma_z^* H_{xz} = \partial z E_y \tag{2}$$

$$\epsilon_0 \frac{d}{dt} E_{xz} + \sigma_z E_{xz} = -\partial z H_y \tag{3}$$

$$\epsilon_0 \frac{d}{dt} E_{yx} + \sigma_x E_{yx} = K_x^m H_z \tag{4}$$

where $K_x^m = \frac{m\pi}{a}$ and $a,\ b$ are cross-sectional dimensions of the waveguide.

Note that the split field components are combined on the right-hand side of the above equations, e.g., $E_y = (E_{yx} + E_{yz})$. Also, E_z and H_z amplitudes are the actual field amplitudes multiplied by $(-j)$ to account for 90° phase shift and allow for real arithmetics.

The field update formulae can be derived next. As an example, equations for $H_{xz}^{n+0.5}$ and E_{yx}^{n+1} are listed below. It is assumed that $\sigma_x^* = \sigma_y^* = \sigma_y = \sigma_x = 0$.

$$\begin{aligned} H_{xz}^{n+0.5}(i,m) &= \alpha_{hz}^1 H_{xz}^{n-0.5}(i,m) \\ &\quad + \alpha_{hz}^2 \left[E_y^n(i+1,m) - E_y^n(i,m)\right] \end{aligned} \tag{5}$$

$$E_{yx}^{n+1}(i,m) = E_{yx}^n(i,m) + K_x(m)\frac{dt}{\epsilon_0} H_z^{n+0.5}(i,m) \tag{6}$$

where i denotes location along z axis (as in the standard FDTD notation) and m denotes mode number.

When exponential differencing is used coefficients α take the following form:

$$\alpha_{hz}^1 = \exp\left(-\sigma_z^* \frac{dt}{\epsilon_0}\right) \tag{7}$$

$$\alpha_{hz}^2 = \frac{1 - \alpha_{hz}^1}{\sigma_z^*\, dz} \tag{8}$$

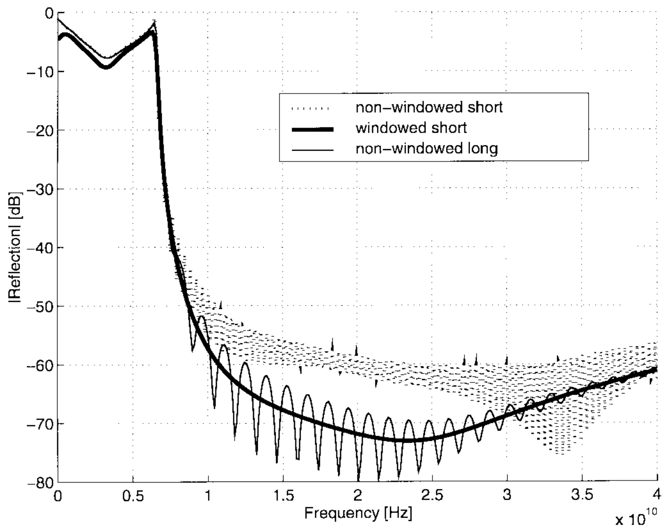


Fig. 1. Effect of signal windowing. PML (6 p1.7–60 dB).

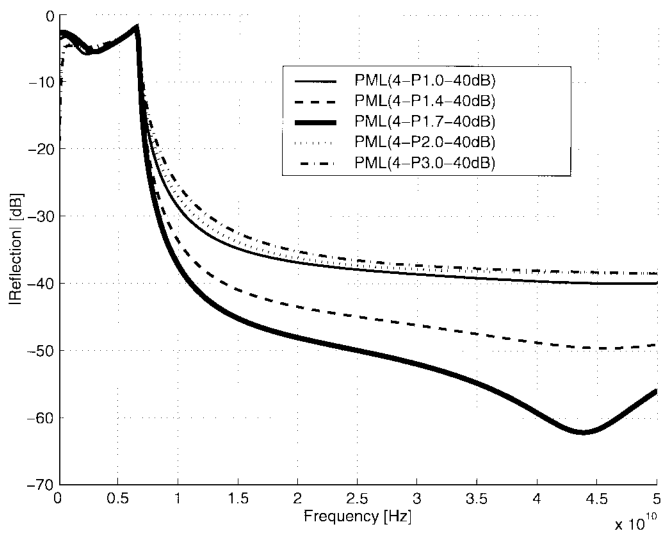


Fig. 2. PML with various power profiles.

whereas for central differencing they are

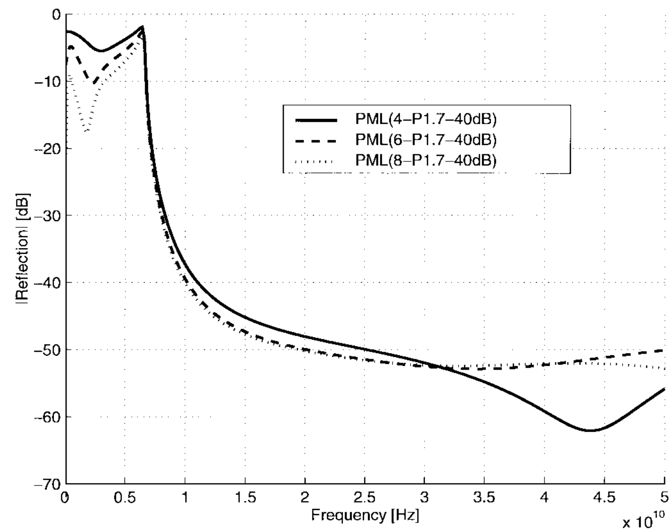
$$\alpha_{hz}^{1} = \frac{1 - \sigma_z^* \frac{dt}{2\mu_0}}{1 + \sigma_z^* \frac{dt}{2\mu_0}} \tag{9}$$

$$\alpha_{hz}^{2} = \frac{1}{\mu_0}\frac{dt}{dz}\frac{1}{1 + \sigma_z^* \frac{dt}{2\mu_0}}. \tag{10}$$

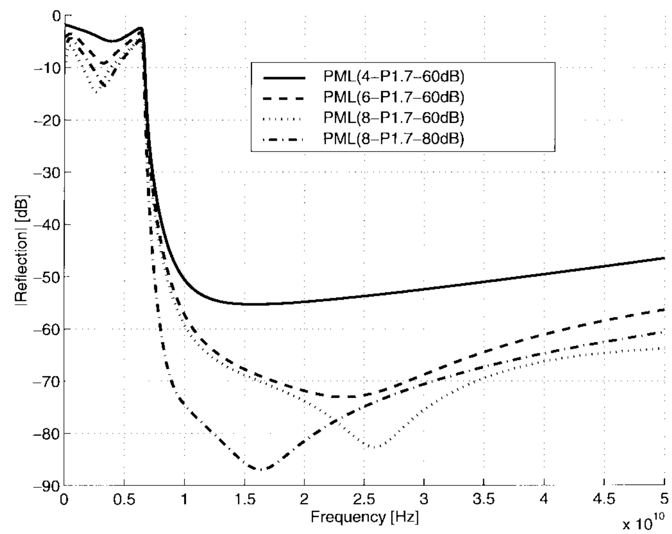
Since typically very few modes have to be considered at the PML interface, PML computations become very efficient. Even PML's with many layers $(N > 10)$ can be used and computed at a negligible fraction of the overall computational expense.

III. Numerical Results

Performance of the modal PML was tested in a rectangular waveguide 22.86 mm $\times$ 10.16 mm excited in the fundamental mode. The excitation was with a Gaussian pulse with a 50-GHz bandwidth. The space and time steps were, respectively, 0.5 mm and 1.6 ps. The reflection coefficients were evaluated at the PML interface.



(a)



(b)

Fig. 3. Response of PML with power profile $p = 1.7$ and various number of layers N and normal reflection R_o.

Fig. 1 shows the reflection coefficient for three cases with and without a windowing algorithm applied prior to the spectral analysis. In the time-domain response of the PML, there are slowly decaying oscillations whose periods corresponds to the cutoff frequency. This is in agreement with the earlier observations [9] that evanescent and cutoff waves are not absorbed by PML media used here. As the cutoff frequency oscillations fade slowly, their abrupt truncation triggers oscillations visible in the spectrum of the reflected signal. The effect of the truncation is prominent in Fig. 1 for both short and long nonwindowed cases. This phenomenon contaminates the results for the computed reflection coefficient. Therefore, in the further analysis, we use the conventional Blackmann window to remove it.

Fig. 2 illustrates the behavior of the reflection coefficient as a function of frequency for a PML with a small number of layers for various p values of the PML power profile defined as

$$\sigma(z) = \sigma_{\max}\left(\frac{z}{dz}\right)^{p}.$$

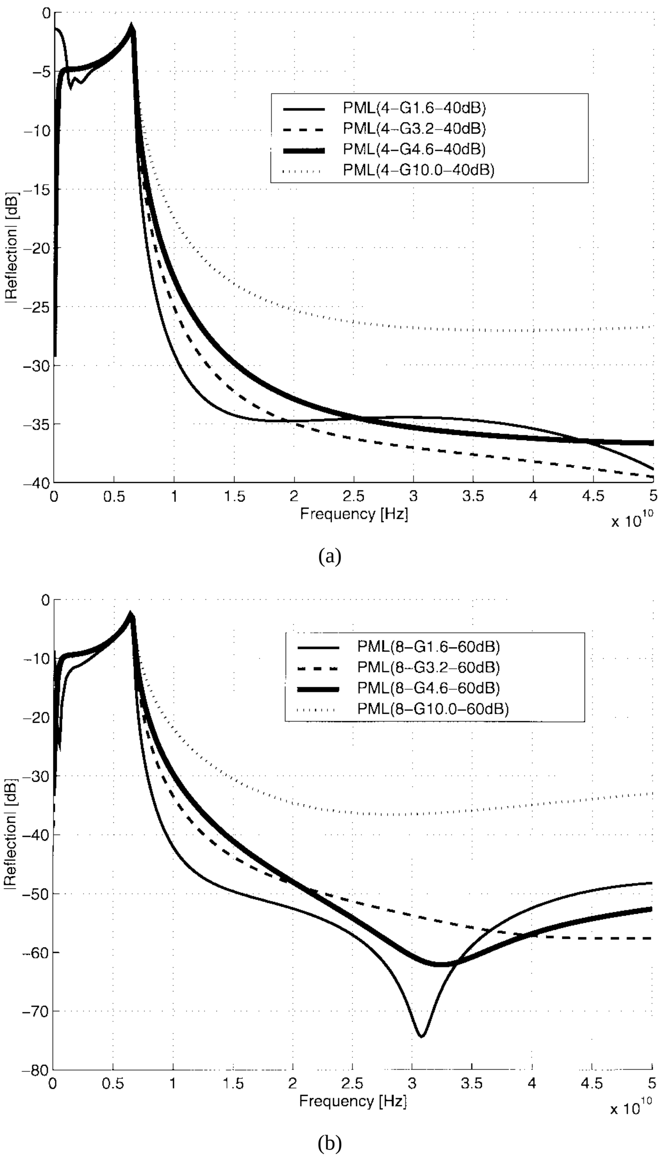


Fig. 4. Response of PML with various geometrical profiles with (a) normal reflection $R_o = -40$ dB, $N = 4$; (b) normal reflection $R_o = -60$ dB, $N = 8$.

Several observations can be made. Large reflection exists below and at the cutoff frequency [9]. The optimum performance in the normal waveguide operation bandwidth and above occurs for $p = 1.7$, with typically more than 10-dB improvement over linear or parabolic profiles. As the frequency increases, this effect disappears and the computed reflection coeffcient tends to the theoretical value R_o. The superior performance for this profile appears to be independent of the number of layers and the maximum reflection at the end of the PML. This observation is based on our investigations for N up to 12 and R_o up to −100 dB. The frequency behavior of the reflection coefficient for various N and R_o is illustrated in Fig. 3 for the optimal $p = 1.7$. Some improvement in the performance can be noticed with an increase in the number of layers and lower reflections at the PML termination, e.g., for $N = 8$ and $R_o = -60$ dB.

Fig. 4 shows the reflection characteristics for two representative sets of N and R_o for PML medium with geometrical profile defined as

$$\sigma(z) = \sigma_o\left(g^{\frac{z}{dz}}\right).$$

Profiles with other N and R_o were also investigated and produced similar results with the overall best performance for $g = 1.6$ and somewhat better performance of PML with g between 3 and 5 at higher frequencies. A comparison of the modal PML with the two conductivity profiles indicates on the overall superior performance of the power profile with $p = 1.7$ (Figs. 2 and 3). However, geometrical profile can provide smaller reflections at very low frequencies.

Our additional tests also have showed that there is no significant difference in the behavior of the modal PML formulated with central or exponential differencing. This property of the modal FDTD is the same as in the earlier Veihl and Mittra report regarding the PML in the FDTD method [10].

IV. Conclusion

A modified PML provides a high-performance termination of open spaces in the hybrid FDTD modal analysis. Application of the windowing technique allows for use of much shorter time series. Our tests also showed that there is no perceivable difference between results obtained with central and exponential differencing of PML equations. We have examined both geometrical and power profiles of PML conductivities. For the configuration analyzed the following observations can be made. The power profile with the coefficient of 1.7 gives the optimal overall performance. Both profiles behave poorly below cutoff, but the geometrical profile shows significant improvement at very low frequencies. The PML was tested in a rectangular waveguide, but it is expected that the modal PML will work well in homogeneous waveguides of other cross sections.

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